STUDIES ON MIXING. XXXI.*
PUMPING CAPACITY OF A TURBINE MIXER**

I. FOŘT, J. DRBOHLAV, J. KRÁTKÝ, M. GROSPIČOVÁ and Z. KROUŽILOVÁ
Chemical Engineering Department,
Institute of Chemical Technology, Prague 6

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A relation was derived for calculating the pumping capacity of a rotating turbine mixer *i.e.* the volume of liquid flowing per unit of time through the imaginary cylinder circumscribed about the blade tips of the rotating impeller, by solving the continuity equation for the mixer region. By use of the obtained equation can be, from knowledge of the mixer geometry and its rotational speed, determined the velocity profile of the flow at the exit from the blades of a turbine mixer and determined its pumping capacity. Experimental verification of the theoretically derived equation was made by measuring the velocity profile of the flow at the exit from the rotating six-blade turbine mixer by using a five-holes Pitot tube in a cylinder with four radial baffles in turbulent flow regime of the mixed charge. Results of these experiments were found to be in good agreement with theoretically derived equations with the mixer size equal to one third and one fourth of the vessel diameter.

The aim of such mixing operation is usually the fastest removal of concentration and temperature gradients in the charge, or for a multiphase system, formation of the largest interphase area. It is therefore advisable to originate a convective flow of turbulent nature in the mixed system so that the flow-rate of any type of the above mentioned operations would be as possible constant in the whole volume of the system. The source of convective flow is the mixer whose pumping capacity is quantitatively expressed by the flow-rate of liquid delivered by the mixer in a time unit. This work is dedicated to the study of pumping capacity of turbine mixers.

Rushton, Mack and Everett[1] used in direct measurements of pumping capacity of a turbine mixer two concentric cylindrical vessels and measured the amount of liquid forced by the mixer from one vessel to the other in a unit of time. Thus they obtained a certain information concerning the pumping capacity of the mixer. The described experimental method is relatively simple, but it considerably affects the flow direction, *i.e.* the flow-pattern in an actual system is rather different from the one in the system in which the authors took their measurements.

Sachs and Rushton[2] studied the velocity field in vicinity of a rotating turbine mixer by use of a photographic method of traces. By evaluating the path of the trace-particles on the photographs they obtained informations on the velocity field in the studied region and from these data

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they calculated by integration the pumping capacity of the turbine mixer. Metzner and Taylor[3] used similarly the photographic method of traces for study of the velocity field in vicinity of the mixer. Unlike Sachs and Rushton[2] they carried out their measurements at values of $Re < 1{\cdot}0 \cdot 10^3$, when the flow regime of the charge cannot be considered turbulent in its whole volume.

The photographic method was also used by Cutter[4], who compared its results with those obtained by the pressure Kiel-probe. He measured the velocity profile of the liquid streaming from the turbine blades and calculated the pumping capacity and from it the flow-rate criterion K_p. It is possible to say, that the photographic method of traces does not interfere with the characteristics of the flow in the given system, but that it is too time-consuming as far as its evaluation concerns; thus it is recommendable at the most as the standard comparison method.

The method of observation of particle motion having the same density as the mixed charge was used by Sato and Taniyama[5] for study of a model flow in the mixed vessel. They derived theoretically the relation between the mean of two consecutive passages of the particle through the region of a rotating impeller (the so-called mean circulation time) and the pumping capacity of the mixer from which they calculated the value of the flow-rate criterion. They have not proved experimentally the correctness of their equation relating their experimental data on the mean circulation time of the indicating particle with the pumping capacity of the turbine mixer. Aiba[6] measured the velocity profile in the liquid flow streaming from the region of a rotating mixer by visual observation of the displacement of a suspended ball (with variable density) which was situated in the flowing liquid. Though this method is quite simple, it is affected with a large error caused by inexact determination of the mean position of the ball in the given point due to quick pulsation of the charge surrounding it. The velocity fields in the mixed vessel was measured by a thermistor probe by Norwood and Metzner[7] in both laminar and turbulent flow regime of the mixed charge. From these results they obtained the equation correlating the dimensionless pumping capacity of the turbine mixer with Reynolds number. The given proportionality factor is dimensional. A film anemometer was used by Oldshue[8] for obtaining the velocity profile of the resulting liquid velocity vector at the exit from the region of the rotating mixer. In this way he obtained not only the profile of the mean velocity but also the profile of mean value of second power of fluctuation velocity in the flow streaming from the mixer blades. Holmes, Voncken and Decker[9] described the method of measuring the local velocity by a miniature propeller. They obtained the velocity distribution in the plane of mixer symmetry along the radial ray. From these results they estimated the volumetric flow-rate of the liquid in the radial direction in different distances from the mixer. The obtained conclusions are, however, affected by a considerable inaccuracy since the used measuring method is subject to a relatively large measuring device in respect to the cross-sectional area of the flow which leaves the blades of the rotating turbine. The measurements of liquid velocity by the method of dynamic pressure acting on the ball was made by Kafarov and Ogorodnik[10]. The measuring and evaluation method is relatively simple but the measuring element similarly with the previously described method, does not satisfy the condition of point measurements.

For measuring the velocity field is very frequently used the method of Pitot tubes. Nagata, Yamamoto, Hashimoto[11] and Nagata, Yamamoto, Ujihara[12] obtained the flow-pattern in the charge. Błaziński and Tyczkowski[13] measured by a spherical five-holes Pitot tube the liquid velocities in several points on the radial ray passing through the centre of the mixer blade. They calculated the flow-rate criterion from experimental values of angles of the liquid flow direction and from absolute values of the velocity vector in the blade axis for different relative sizes of the turbine mixer. Wolf and Manning[14] used the Pitot–Prandtl tube for measuring the velocity profile in vicinity of the turbine mixer. This method enables to determine the magnitude of the velocity vector simultaneously with the static pressure, but the authors did not mention these quantities in their work. Cooper and Wolf[15] used for measurements of the velocity field in the liquid flow leaving the blades of the mixer a three-holes and a five-holes Pitot tube and they verified the results obtained by measurements with the hot wire anemometer in air. From the results of experiments they determined then the magnitude of the flow-rate criterion.

From the cited results follows that values of the flow-rate criterion K_p in turbulent region of flow of the mixed charge ($Re > 1 \cdot 0 . 10^4$) obtained by different experiments greatly differ (Table I).* This is caused primarily by use of different experimental techniques and secondly by different geometrical arrangement of the mixed system. This fact will be further on taken into account in comparison of different results with those presented here.

Van de Vusse[16] considered in his work that the turbine mixer causes, in a system with baffles, a flow similar to that one of liquid leaving the blades of a centrifugal pump and he obtained relation for the pumping capacity of the turbine mixer in the form

$$\dot{V} = 4\pi^2 n d^2 h \sqrt{(1 - q^2)^{1/2}} . \qquad (1)$$

From this result follows that the pumping capacity of a turbine mixer is proportional to rotational speed of the mixer, to the second power of its radius and to the blade width. The proportionality constant includes a quantity q, having a value which is known to be within the range between zero and one as it gives the ratio ω'/ω where the angular liquid velocity ω' is lower than the angular velocity ω of the mixer blades. From the assumptions made by the author[16] follows, however, that the real value q is rather uncertain and is furthermore a function of the vertical distance from the horizontal plane passing through the centre of the mixer.

Cooper and Wolf[15] have chosen an analytical method for obtaining the relation between the pumping capacity of the turbine mixer and its dimensions. In principle they are balancing material and momentum over a part of the region of a rotating impeller which is represented by a hollow cylinder with the outside diameter equal to the mixer diameter d, inside diameter Kd and the height Δz, with $K \in (0; 1/2\rangle$. Further on, this region is called the annular region of the mixer. The authors have made several assumptions: *1.* The cylindrical region of diameter Kd, which is an extension of the annular region of the mixer, rotates with an angular velocity ω, *i.e.* with the angular velocity of the mixer; *2.* relations $w_{ax} = \text{const.}$ are valid if $z = z$ and $r \in (Kd/2, d/2\rangle$, $w_{ax} = 0$ if $z = z$ and $r < Kd/2$; *3.* $w_{rad} = 0$ is valid for $r \leqq Kd/2$; *4.* the angular liquid velocity ω' in the whole region of the totating mixer equals to the angular velocity of its blades ω; *5.* $w_{ax} = 0$ is valid for $z = 0$ and $r = d/2$; *6.* the mixed charge is considered to be incompressible.

By solving the material and momentum balances across the annular region of the mixer at the given assumptions, the authors[15] have obtained the relation for the profile of radial liquid velocity component in dependence on the vertical or axial coordinate z at the point $r = d/2$. By integration of the obtained velocity profile over the external shell of an imaginary cylinder circumscribed to the blades of the rotating mixer, they obtained an equation for the pumping mixer capacity in the form

$$\dot{V} = 2\pi d^3 n k'^3 \ln \{ \sin [(h/2c^2 d) + (\pi/4)]/\sin \pi/4 \} . \qquad (2)$$

From this equation follows that the pumping capacity of the turbine mixer depends on the first power of the rotational speed and on the third power of the mixer diameter, on the ratio h/d, while the constant $k' \in \langle 0.614; 0.707 \rangle$. The cited derivation of pumping capacity of the turbine mixer was described in such complete form in literature for the first time. Our opinion is however that even if the results concerning the pumping capacity obtained theoretically after introduction

* From results of the cited papers follows that K_p is independent of Re for $Re > 1 \cdot 0 . 10^4$.

of empirical correction by the authors do agree with the results of experiments, not all assumptions and steps made in derivation are correct and it is necessary to take this fact into account: The proposed solution has its first shortcoming in the assumption No 4. From validity of this assumption would follow that tangential velocity component in the stream leaving the blades of the rotating mixer, would equal to the peripheral velocity of blade tips of the mixer. But actually, due to the centrifugal force, the mentioned velocity component is changed into a radial component and thus only the vector sum of radial and tangential components equals at the most to the peripheral velocity of the blade tips of the mixer. This shortcoming is quite obvious also from the experimental results. For this reason, the authors[15] had to introduce the above mentioned correction factor.

The momentum balance over the region of a rotating mixer cannot be made in the way presented in the cited work[15]. Since balancing of the vector quantity is concerned, the presented method is incorrect due to principles of the vector calculus (for example addition of vectors or addition of vector components). Another disadvantage is that the function $w_{rad} = w_{rad}(z)$ for $r = d/2$ over the interval $\langle -h/2; h/2 \rangle$ has a singular point with the coordinate $z = 0$. For determination of the volumetric capacity the liquid flow rate across one half of the shell width circumscribed to the mixer blades must be calculated and multiplied by 2. Such solution can be then used only for the symmetrical profiles of the radial velocity component across the blade width which means a certain special limitation. Besides, as follows from the experiments made by the authors of this paper[15], the existence of a singular point does not correspond to reality, *i.e.* the profile of radial velocity component across the width of the turbine blade is a continuous function.

TABLE I

Published Results of Experimentally Determined Pumping Capacities of a Six-Bladed Turbine Mixer ($h_2/D = 0{\cdot}3$)

Author	d/D	K_p
Cutter[4]	0·348	1·72
Cooper[15]	0·267	0·79
Blasiński and Tyczkowski[13]	0·140—0·500	1·08
Norwood and Metzner[7]	0·167—0·500	1·21
Souza and Pike[17]	0·200	0·887
Souza and Pike[17]	0·250	0·793
Souza and Pike[17]	0·267	0·926
Souza and Pike[17]	0·333	1·023
Souza and Pike[17]	0·400	1·189
Sato and Taniyama[5]	0·200[a]	0·624
Sato and Taniyama[5]	0·250[a]	0·703
Sato and Taniyama[5]	0·333[a]	0·810
Sato and Taniyama[5]	0·400[a]	0·885

[a] $h_2/D = 0{\cdot}5$.

It can be therefore summarized that efforts to derive analytically the dependence of pumping capacity of the turbine mixer on geometrical parameters of the mixer and its rotational speed is interesting and useful for suggestion of a successful way to obtaining a correct relation. But because of the mentioned objections, it will be necessary to subject the studied problem to further analyses.

Souza and Pike[17] made a successful attempt with solving the equations of turbulent flow for the leaving the flow region of the turbine mixer as a tangential jet. A solution of the Reynolds equation was thus obtained in the form of a three-parameters equation when the mentioned parameters must have been calculated from experimentally obtained profiles $w_{rad} = w_{rad}(z)$. We see the shortcoming of this solution in quite different dimensions of the considered tangential jet than those of the used mixer. Three parameters of the solution cannot be therefore clearly physically interpreted and consequently their value cannot be estimated only on the basis of knowledge of the mixer dimensions and of the mixed system, but it is necessary, as presented by the mentioned authors, to determine correlations between the mentioned parameter and quantities which characterize the geometrical relations of the system.

THEORETICAL

The pumping capacity or volumetric flow-rate of the turbine mixer is considered to be the liquid volume flowing per a unit of time through the shell of an imaginary cylinder circumscribed to the blade tips of a rotating mixer. This cylindrical region will be hereinafter called the region of a rotating mixer. The pumping capacity of the mixer related to the product of the first power cf the rotational speed of the mixer and to the third power of its diameter defines the so-called flow-rate criterion

$$K_p \equiv \dot{V}/nd^3 . \qquad (3)$$

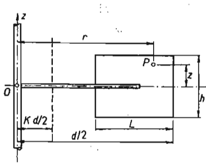

FIG. 1
Rotor Region

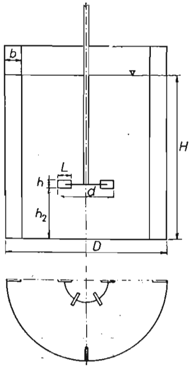

FIG. 2
Schematic View of the Mixed System with a Turbine Mixer

We shall concern ourselves to the derivation of the relation between the pumping capacity and the geometrical and kinematic parameters of the turbine mixer. The system in which our study is made is understood the rotor region formed by a hollow cylinder with the outside diameter d (the mixer diameter) and inside diameter Kd, where

$$K \in \langle 0\,;\; 1) \,. \tag{4}$$

The upper base is formed by a horizontal plane passing through the coordinate $z = z_2$ and the lower base by the horizontal plane passing through the coordinate $z = z_1$ when it holds

$$z_2 - z_1 \leqq h \,. \tag{5}$$

Both the schematic view of the rotor region and of the mixer are given in Fig. 1 together with indicated positions and orientation of the coordinate axes. It is a cylindrical system symmetrical to the z axis. Coordinates of an arbitrary point P in the rotor region are thus z and r.

Our task is the solution of a continuity equation of incompressible liquid for in this way defined rotor region. We introduce the following simplifying assumptions: *1.* Liquid flows into the region of a rotating turbine mixer only through bases of the rotor region and is discharged by it only through the external shell of this region; *2.* liquid in the cylindrical region with diameter equals to the inside diameter of the rotor region Kd is stationary and behaves as a solid body. This means that through the inside shell of the rotor region is not passing any mass; *3.* axial velocity component of the liquid w_{ax} inside the rotor region is only a function of vertical (axial) coordinate z; *4.* radial velocity component of the liquid w_{rad} inside the rotor region is a function of vertical (axial) coordinate z as well as of the radial (horizontal) coordinate r; *5.* derivation of axial velocity component according to variable z is inside the rotor region only a function of this variable and has the form

$$\mathrm{d}w_{ax}/\mathrm{d}z = \omega(cz^2 + bz + a)\,, \tag{6}$$

where c, b, and a are constants for the given geometrical arrangement of the mixed system.

The continuity equation given in cylindrical coordinates for the axially symmetrical system at a steady state is[18]

$$\frac{1}{r}\,\frac{\partial(\varrho r w_{rad})}{\partial r} + \frac{\partial(\varrho w_{ax})}{\partial z} = 0\,, \tag{7}$$

which is transformed for the rotor region filled with incompressible charge at the considered validity of assumptions *3.* and *5.*, into the form

$$\frac{1}{r}\frac{\mathrm{d}(rw_{\mathrm{rad}})}{\mathrm{d}r} = -\,\omega(cz^2 + bz + a)\,, \tag{8}$$

where the variable z is considered to be a parameter. Eq. (8) is solved by separation of variables and by integration according to r in the interval $\langle Kd/2;\, d/2\rangle$. By further modification we obtain this solution in the form

$$w_{\mathrm{rad}} = -\,\omega(d/2)\,k(cz^2 + bz + a)\,, \quad [r = d/2]\,, \tag{9}$$

$$\text{where} \quad k = 1/2(1 - K^2)\,. \tag{10}$$

The radial velocity component is transformed into a dimensionless form by the relation

$$W_{\mathrm{rad}} = w_{\mathrm{rad}}/(\pi dn) \tag{11}$$

with the double of axial coordinate z related to the blade width h by

$$Z = 2z/h \tag{12}$$

and by introducing

$$C = ckh^2/4\,, \quad B = bkh/2\,, \quad A = ak \tag{13}$$

Eq. (9) is then, with regard to definitions (11)–(13) getting the form

$$W_{\mathrm{rad}} = CZ^2 + BZ + A\,, \quad [r = d/2]\,, \tag{14}$$

where constants A, B and C must be determined from experimentally determined profile $W_{\mathrm{rad}} = W_{\mathrm{rad}}(Z)$.

The physical significance of parameters of Eq. (14) is further considered from the vertical (axial) profile of dimensionless radial velocity component in the flow leaving the rotor region. For the maximum value of the velocity in the considered profile holds

$$\mathrm{d}W_{\mathrm{rad}}/\mathrm{d}Z = 0\,, \quad [r = d/2]\,, \tag{15}$$

$$\text{so that} \quad 2CZ_{\max} + B = 0\,. \tag{16}$$

The value of coordinate where the quantity W_{rad} attains the maximum value $(W_{\mathrm{rad}})_{\max}$ is

$$Z_{\max} = -\,B/2C\,, \quad [r = d/2]\,, \tag{17}$$

while its magnitude is

$$(W_{\mathrm{rad}})_{\max} = A - B^2/4C\,, \quad [r = d/2]\,. \tag{18}$$

For coordinates of points Z_1 and Z_2 on the velocity profile where the radial component of the velocity reaches the zero value holds

$$CZ^2 + BZ + A = 0\,, \quad [r = d/2] \tag{19}$$

so that the values of sought coordinates are

$$Z_{1,2} = [-B \pm (B^2 - 4AC)^{1/2}]/2C\,, \quad [r = d/2]\,, \tag{20}$$

while coordinates Z_1 and Z_2 cannot be complex values because due to Eq. (*18*) the value of discriminant in Eq. (*20*) is always positive.

From the mentioned analysis follows that parameters A, B, C of the velocity profile (*14*) determine the relation between the maximum value of a dimensionless radial component of the liquid velocity, the axial shift of the axis of liquid flow leaving the rotor region, in respect to horizontal plane of the blade symmetry and the width of this flow.

The pumping capacity of the turbine mixer can be determined by integration of the radial velocity component across the width $Z_2 - Z_1$ of the liquid flow streaming from the blades along the mixer periphery, from equation

$$\dot{V} = 1/2\pi^2 nd^2 h \int_{Z_1}^{Z_2} (CZ^2 + BZ + A)\,\mathrm{d}Z\,. \tag{21}$$

After integration we obtain the relation for calculation of the flow-rate criterion

$$K_p = 1/2\pi^2 h/d[C/3(Z_2^3 - Z_1^3) + B/2(Z_2^2 - Z_1^2) + A(Z_2 - Z_1)]. \tag{22}$$

From Eq. (*22*) follows that the flow-rate criterion is dependent on the dimensions of the turbine mixer and on parameters of the velocity profile which must be determined experimentally.

EXPERIMENTAL

The experiments were carried out in a cylindrical vessel of inside diameter $D = 1\,000$ mm (Fig. 2). The vessel was provided with four radial baffles extended to the bottom of the width $b = D/10$. The vessel was filled with distilled water up to the height $H = D$. The whole cylindrical vessel was situated into a cubic vessel with length of the edge 1 200 mm. The space between the two vessels was also filled with water. It supported the wall of the cylindrical mixing vessel and insulated it to the surroundings. For mixing was used a six-bladed turbine mixer of diameter $d = D/3$ and $d = D/4$. Characteristic dimensions of the mixer are in the ratio $d : h : L = 20 : 4 : 5$. The mixer was always situated at the vessel axis and the distance of the lower blade edge from the bottom was $h_2 = 0{\cdot}3D$. The mixer was driven by a direct-current motor with an input 3·5 kW; rotational speed of the motor was adjusted by a gear-box and was kept at a fixed value by an electromagnetic controller.

The five-holes Pitot tube used as the measuring element for determination of absolute values and of vector direction of local, in time mean velocity, was designed as to comply with most of requirements set for such a device[19]. The tube had the form of a regular rectangular frustum of pyramid with the walls forming an apex angle 90°. In each of the four bevelled walls was bored a hole of 0·47 mm diameter. In the axis of the tube was made the fifth hole of the same diameter, with a conical recess with an apex angle 60°. The schematic view of the tube together with its dimensions and with marking of the holes used in this work is given in Fig. 3. The holes of the tube were interconnected by stainless steel tubes of 0·8 mm diameter and were further connected by polyethylene hoses to inclined manometers filled with water. Fixing of the tube and its handling was enabled by a clamping device which consisted of the tube holder, travelling and slewing device. The calibration, determination of calibration dependences and the calibration equipment of the used five-holes Pitot tube is described in detail in our previous work[19].

The measurement of velocity profile of the flow leaving the rotor region was made in the vertical (axial) ray of radial coordinate r (Fig. 1) which was by 1 mm greater than the mixer radius $d/2$. Individual measurements, *i.e.* readings of pressures on manometers, were made at tube locations 3 mm apart from each other in the direction of vertical axis z across the whole width h of the mixer blade. In measurements with the mixer in operation (in measurements with the mixer at rest were obtained referential data) was assumed that the pressure reading on the manometer is steady if two consecutive measurements made in a 5-minute interval did not differ by more than was the accuracy of the manometer reading. Since the used tube has finite dimensions, the data measured by its end holes were graphically corrected to the position of the middle hole

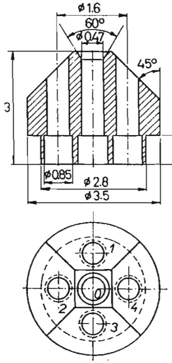

FIG. 3

Five-Holes Pitot Tube

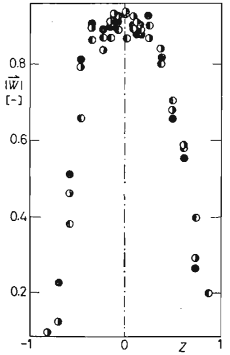

FIG. 4

Profile of Absolute Value of Local Velocity Vector in the Stream Leaving the Rotor Region ($d/D = 1/4$)

◐ n 100 min^{-1}; ◑ n 150 min^{-1}, ● n 200 min^{-1}.

thus 5 pressure data were obtained in a position given by the intersection of the probe axis and of the measured vertical ray.

Independent variables were measured with the following accuracy: The rotational speed of the mixer was kept at a set value with an accuracy $\pm 0{\cdot}5$ r.p.m., distance h_2 of the mixer above the vessel bottom and height H of the mixed liquid surface in the vessel were measured by a measuring scale with an accuracy $\pm 1{\cdot}0$ mm. Diameters of the used mixers were determined in the manufacture with an accuracy $\pm 0{\cdot}1$ mm. The tube location in the mixed system was fixed by a movable device with an accuracy of adjustment in both coordinates r and $z \pm 0{\cdot}25$ mm. Turning of the tube axis, defined by angle α_2 (angle between the tube axis and the vertical plane) and β_2 (angle between the tube axis and the horizontal plane) was carried out with an accuracy $\pm 1°$. The temperature of the mixed charge was kept constant at $18 \pm 2°C$.

The dependent variables were in the experiments the pressure differences Δp_i [$i = 0, 1, 2, 3, 4$], obtained by subtraction of the so-called reference position (*i.e.* the values of the manometers reading with the mixer at rest) from the value of over-all pressure measured by the manometer connected with the given hole. Pressure differences were measured with an accuracy $\pm 2{\cdot}5$ N/m^2, eventually, at fluctuation of the measured pressures, with an accuracy up to $\pm$ 15N/m^2 while the relative accuracy of the measured pressure differences did not differ for more than 5% from the measured value. Absolute value of the local, in time mean velocity, calculated from the measured pressure differences Δp_i [$i = 1, 2, 3, 4, 5$] was then affected by an error arising from accuracies of determination of pressure differences and by the error which is the result of neglecting the second root of mean value of the fluctuation velocity component to its time mean value[19]. Table II gives the dependence of the relative error $\Delta w/\overline{w}$ of absolute value of the local mean time velocity vector on magnitude of this absolute value as follows from the calculation procedure of this quantity from the measured pressure differences Δp_i[19]. Table III then gives the dependence of the same relative error on value of turbulence intensity $(\overline{w'^2})^{1/2}/\overline{w}$. From this table also follows that for the turbulence intensity lesser than 50%, which is a limitation valid for the flow leaving

TABLE II

Dependence of Relative Error of Absolute Value of Local, Mean Time Velocity Vector on Value of this Quantity

w, m s^{-1}	0·2	0·3	0·4	0·5	0·6	0·7	0·9	1·0	1·2	1·4	1·6
$\Delta w/\overline{w}$, %	±40·2	±17·9	±10·0	±6·5	±4·5	±4·2	±5·0	±4·6	±4·4	±3·8	±3·8

TABLE III

Effect of Turbulence Intensity on Accuracy of Determination of Absolute Value of Local, Mean Time Velocity Vector

$(\overline{w'^2})^{1/2}/\overline{w}$, %	10	20	30	40	50
$\Delta w/\overline{w}$, %	0·5	1·8	4·2	7·0	10·5

the rotor region[8], is the error, caused by neglecting the fluctuation component in respect to the mean time value, less than 10%, while the mean value of turbulence intensity in the studied flow is less than 30%. With regard to this and with regard to the found absolute value of the local mean time velocity vector, the relative error of this quantity, $\Delta w/\overline{w}$, for our measurements is within the interval $\Delta w/\overline{w} \in \langle 4\%; 10\% \rangle$. From this fact follows that the accuracy of determination of all independent variables was considerably greater than the accuracy of determination of the dependent variable, *i.e.* of the absolute value of the local velocity vector.

RESULTS

From results of measurements of pressures by the five-holes Pitot tube were successively determined the absolute value of the local mean time velocity vector and of the local in time mean value of static pressure* by the procedure described earlier[19]. Further was determined the direction of the flow at the given point, given by the angle between the local velocity vector and the horizontal plane (angle β) and the vertical plane, in which is situated the axis of symmetry (angle α). Further were calculated the values of radial, tangential and axial components of local velocity with the use of relations

$$\overline{w}_{rad} = \overline{w} \cos \beta \cos \alpha \,, \tag{23a}$$

$$\overline{w}_{tg} = \overline{w} \cos \beta \sin \alpha \,, \tag{23b}$$

$$\overline{w}_{ax} = \overline{w} \sin \beta \tag{23c}$$

and transformed into a dimensionless form with the use of Eq. (*11*) and of the relations

$$W = \overline{w}/(\pi \, dn) \,, \tag{24a}$$

$$W_{tg} = \overline{w}_{tg}/(\pi \, dn) \,, \tag{24b}$$

$$W_{ax} = \overline{w}_{ax}/(\pi \, dn) \,. \tag{24c}$$

The measured local static pressure was also transformed into a dimensionless form with the use of equation

$$P_{st} = p_{st}/[\varrho(\pi \, dn)^2] \,. \tag{25}$$

TABLE IV

Parameters of Velocity Profile $W_{rad} = W_{rad}(r)$ at the Exit from the Rotor Region

d/D	A	σ_A	B	σ_B	C	σ_C	K_p	σ_{rel}, %
1/4	0·669	0·011	0·026	0·020	−1·111	0·041	0·70	5·0
1/3	0·640	0·015	0·210	0·030	−1·068	0·068	0·76	6·2

* Hereinafter will always be by local velocity understood the local mean time velocity. This applies analogically to the static pressure.

In Figs 4–13 are given the vertical profiles of dimensionless quantities W, W_{rad}, W_{ax}, P_{st} and angle α in the flow leaving the rotor region for the used mixer sizes while as a coordinate determining the position is considered the dimensionless vertical coordinate Z, defined by Eq. (*12*).

From the measured vertical profile $W_{rad} = W_{rad}(Z)$ were calculated by the method of least squares the constants A, B and C of parabola (*14*). The mentioned constants are together with estimation of their standard deviations given in Table IV. From thus calculated parameters of the profile of quantity W_{rad} it was possible with the use of Eq. (*22*) to calculate the values of flow-rate criteria for the used mixer sizes. They are also given in Table IV together with estimate of their relative standard deviations determined from estimates of standard deviations of the constant A, B and C.

DISCUSSION

Profile of Absolute Value of the Local Velocity Vector and of the Direction of Flow Leaving the Rotor Region

The vertical profile of absolute value of the local velocity vector in the flow leaving the rotor region (Fig. 4 and 5) has similar courses for the both used mixer sizes: the greatest velocity of the said flow is in vicinity of the symmetry plane of the mixer and it descends rather sharply towards the blades tips. Since the experiments were

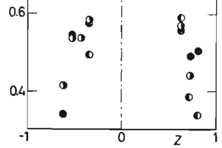

FIG. 5

Profile of Absolute Value of Local Velocity Vector in the Stream Leaving the Rotor Region ($d/D = 1/3$)

◐ n 50 min^{-1}; ◑ n 75 min^{-1}; ● n 100 min^{-1}.

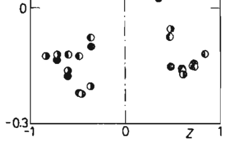

FIG. 6

Profile of Static Pressure in the Stream Leaving the Rotor Region ($d/D = 1/4$)

◐ n 100 min^{-1}; ◑ n 150 min^{-1}; ● n 200 min^{-1}.

carried out in turbulent flow regime of the charge ($Re > 9{\cdot}0 \,.\, 10^4$), profile $W = W(Z)$ is independent of the rotational speed of the mixer but it depends significantly on its size. With the mixer of a relative size $d/D = 1/3$, the asymmetry of the mentioned profile becomes apparent, while in the flow leaving the rotor region of the mixer with the relative size $d/D = 1/4$ the velocity profile along the mentioned plane is symmetric. Explanation of this fact is obviously related with the asymmetric position of the mixer in the vessel along the vertical axis, since the volume of the liquid above the symmetry plane of the mixer is twice as large as the volume of the charge below this plane. But asymmetry of the profile $W = W(Z)$ is far smaller than ratio of the mentioned volumes and with the mixer of a relative size $d/D = 1/4$ the profile can be considered symmetric along the symmetry plane of the blade.

The absolute value of local velocity vector expressed as a ratio of peripheral velocity of the tips of the mixer blades does not reach even at the point of maximum a value equal to unity. This fact is related with the course of function $P_{st} = P_{st}(Z)$ (Fig. 6 and 7). The static pressure attains in vicinity of the point where function $W = W(Z)$ reaches the maximum, positive values. This has to be taken into consideration in calculation of specific energy of the flow leaving the rotor region at the point $Z = Z_{max}$. Here, at the assumption that turbulence in vicinity of the mentioned point can be considered isotropic, the measured static pressure can be taken as specific kinetic energy of fluctuation movement[20]. Then we can – with regard to relations (*24a*) and (*25*) – write

$$W^2 + P_{st} = 1\,, \quad [Z = Z_{max}]\,, \tag{26}$$

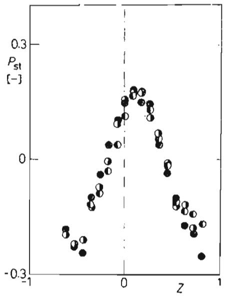

FIG. 7

Profile of Static Pressure in the Stream Leaving the Rotor Region ($d/D = 1/3$)
◐ n 50 min^{-1}; ◑ n 75 min^{-1}; ● n 100 min^{-1}.

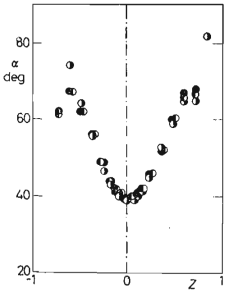

FIG. 8

Profile of Angle α in the Stream Leaving the Rotor Region ($d/D = 1/4$)
◐ n 100 min^{-1}; ◑ n 150 min^{-1}, ● n 200 min^{-1}.

which satisfies very well the determined absolute value of the local velocity vector and local static pressure in the point of maximum profile $W = W(Z)$. This fact confirms the improper assumptions concerning the equality of peripheral velocities of the mixer blade tips and of absolute value of the velocity vector at an arbitrarily chosen point of the flow leaving the rotor region (or any point of this flow), or of the angular velocity of the mixer and liquid in the rotor region[15]. The validity of Eq. (*26*) cannot be, however, extended to the whole interval $Z \in \langle -1 ; 1 \rangle$, since due to losses caused by friction in the rotor region it would be necesary for individual streamlines (with the exception of streamlines passing through the point $Z = Z_{max}$) to extend the left-hand side of the said equation by a term expressing energy losses caused by friction in the rotor region along the considered streamlines. Further, the validity of Eq. (*26*) includes implicitely the following fact; the liquid flow entering the rotor region through its bases (Fig. 1) can be considered stationary in respect to the flow leaving the rotor region. This fact has already been proved experimentally[15].

The course of quantity characterizing the direction of the local velocity vector in the studied flow (angle α) has in vicinity of the symmetry plane of the blade a sharp minimum (Fig. 8 and 9). For a purely radial flow direction, the value of angle α would equal to zero, for the purely tangetial direction it would equal to 90°. From the mentioned figures can be seen that inside the ray streaming from the rotor region, prevails the due to radial velocity component of rotating blades, centrifugal force while towards the blade tips increases the contribution of tangential component as

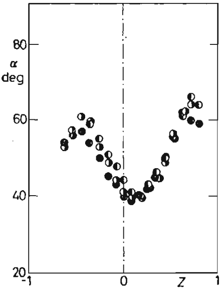

FIG. 9

Profile of Angle α in the Stream Leaving the Rotor Region ($d/D = 1/3$)
◐ n 50 min^{-1}; ◑ n 75 min^{-1}; ● n 100 min^{-1}.

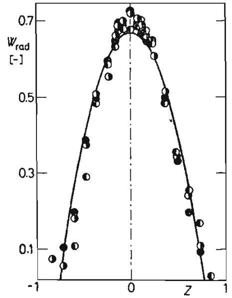

FIG. 10

Profile of Radial Component of Local Velocity Vector in the Stream Leaving the Rotor Region ($d/D = 1/4$)
◐ n 100 min^{-1}; ◑ n 150 min^{-1}; ● n 200 min^{-1}.

in this direction decreases the centrifugal force acting on the liquid. The course of the mentioned profile becomes independent on the rotational speed of the mixer when its minimum attains for both the used mixer sizes practically the same value, but always in different points along the blade width.

Profiles of Components of Local Velocity Vector in the Flow Leaving the Rotor Region

In Figs 10 and 11 are apart from experimental results also plotted the calculated profiles of the radial velocity component (Eq. (*14*)) in the liquid flow leaving the rotor region. From these figures is obvious suitability of the proposed regression function, *i.e.* of the parabola of second degree. Another form of polynomial expressing the vertical profile of radial component of local velocity vector in the flow leaving the rotor region is not suitable even for physical reasons: the linear profile would reach at the point of maximum value the singularity which is not in agreement with the experimental results, since the found profile is a smooth line. The polynome giving the velocity course as a parabola of a third degree is then an odd function, *i.e.* the value of radial velocity component would change its sign in vicinity of the symmetry plane of the mixer blade which is also not in agreement with the experiments. The parabola of the fourth degree has five constants whose physical interpretation with the use of geometrical parameters of the mixer is not possible.

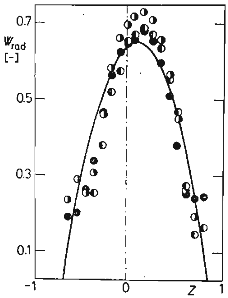

FIG. 11

Profile of Radial Component of Local Velocity Vector in the Stream Leaving the Rotor Region ($d/D = 1/3$)

◐ n 50 min^{-1}; ◑ n 75 min^{-1}; ● n 100 min^{-1}.

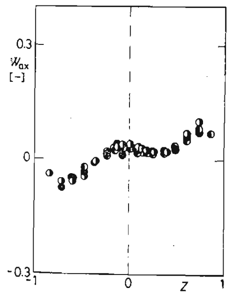

FIG. 12

Profile of Axial Component of Local Velocity Vector in the Stream Leaving the Rotor Region ($d/D = 1/4$)

◐ n 100 min^{-1}; ◑ n 150 min^{-1}; ● n 200 min^{-1}.

The width of the flow leaving the rotor region is lesser than the width of the mixer blades. This means that for the pumping effect of the mixer only its inside part along the horizontal symmetry plane of the blades is used while in vicinity of the upper ($Z = 1$) and lower ($Z = -1$) blade edges prevails the tangential component caused by the rotating mixer as well as the axial component caused by the suction effect of the mixer in the space above and below it. This fact could be taken into account for example in design of the mixer blade with smaller form resistance than has the mixer used in this work, while the pumping effects of such mixer would not change with this modification. The determined vertical profile of the radial velocity component can be, for a mixer of relative size $d/D = 1/4$, considered symmetrical along the symmetry plane of the blade (the value of constant B in Eq. (*14*) is for this mixer because of its variance negligible); for the mixer of relative size $d/D = 1/3$ the shifting of the maximum position of the function $W_{rad} = W_{rad}(Z)$ in respect to the point $Z = 0$ (see Eq. (*17*)) is

$$Z_{max} \approx 0{\cdot}100\,. \tag{27}$$

Vertical profiles of axial component of the local velocity vector in the flow leaving the rotor region (Fig. 12 and 13) have very low values of the said component – in dimensionless form it is lesser than 0·20. From comparison of this quantity with the remaining two components of the local velocity vector follows that the flow from rotor region prevails in horizontal direction and consists from tangetial and radial velocity components. Apart from that, the profile of axial velocity component on the boundary of rotor region can be approximately expressed by a cubic parabola as follows from assumption *5*, *i.e.* by equation

$$w_{ax}(z) = \omega(cz^3 + bz^2 + az + e)\,, \quad [r \in \langle Kd/2;\ \ d/2\rangle]\,. \tag{28}$$

This confirms suitability of the chosen polynome by which is expressed with the accuracy of a constant the profile $w_{ax} = w_{ax}(r)$ and the profile $w_{rad} = w_{rad}(z)$.

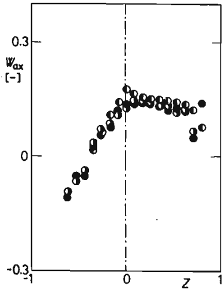

FIG. 13

Profile of Axial Component of Local Velocity Vector in the Stream Leaving the Rotor Region ($d/D = 1/3$)
◐ n 50 min^{-1}; ◑ n 75 min^{-1}; ● n 100 min^{-1}.

Pumping Capacity of the Turbine Mixer

The results of determination of pumping capacity of the turbine mixer, as expressed by use of the flow-rate criterion K_p given in Table IV are to a small extent affected by the relative size of the mixer: with increasing value of the ratio d/D increases as well the value of the flow-rate criterion. This dependence is also in agreement with three of the cited papers[5,15,17] (Table I), even if the extent of this effect is not in these compared papers the same. Absolute values of criterion K_p, determined in this work, are in a very good agreement with the results of authors[5], while certain systematic deviations in the positive direction in papers[15,17] are obviously caused by the chosen expression for dependence $W_{rad} = W_{rad}(Z)$, which is plotted through the experimental data and which reaches in vicinity of points with coordinates $Z = \pm 1$ positive and significantly non-zero value. Then also values of the pumping capacity and of the flow-rate criterion are slightly greater than the values obtained in this work. The values of the flow-rate criterion given in other papers[7,13] differ significantly from ours. The reason for that may be that in the cited works are given values K_p where each of them represents an average from a set determined in a wide interval of ratios d/D, even if the flow-rate criterion significantly changes (non-linearly) in dependence on this ratio. Furthermore, the authors[13] made their measurements only on the radial ray situated in the horizontal plane of symmetry of the mixer blades. On basis of the course of profile $w_{rad} = w_{rad}(z)$ at the point $r = d/2$, published by Sachs and Rushton[2], they then determined the mean value of the radial component across the flow leaving the rotor region for calculation of the pumping capacity.

Cooper and Wolf[15] mention in their work that the flow at the exit from the rotor region can be studied with air as the model liquid by the method of local velocity measurement with the hot-wire anemometer while in water this method cannot be used. Data from the thesis of the former of the authors[15] have been subjected to a statistical analysis: whether for geometrically similar mixed systems in turbulent regime of the charge the results of measurements of velocity fields expressed by the velocity profiles and by the flow-rate criterion with water and with air are the same. It has been found out that the said results are statistically significantly different, *i.e.* neither parameters of the velocity profile nor values ot the flow-rate criteria are in the two mentioned media, for practically identical mixed systems, the same. It is, therefore, necessary to carry out the experiments for determination of the velocity field in vicinity of the turbine mixer in a liquid, since similarity of velocity fields is not valid in mixed system where liquid and gas are used.

The author wishes to thank Dr V. Kudrna, Chemical Engineering Department, Institute of Chemical Technology, Prague, for valuable comments and remarks in theoretical part of this work.

LIST OF SYMBOLS

A	constant defined by Eq. (*13*)
a	constant in Eq. (*9*)
B	constant defined in Eq. (*13*)
b	radial width of baffle [m]
	constant in Eq. (*9*) [m^{-1}]
C	constant defined in Eq. (*13*)
c	constant in Eq. (*9*) [m^{-2}]
D	vessel diameter [m]
d	mixer diameter [m]
e	constant in Eq. (*28*)
H	height of liquid surface from vessel bottom with mixer at rest [m]
h	blade width [m]
h_2	height of lower edge of mixer blade from vessel bottom [m]
K	constant determining the inside diameter of rotor region as multiple of mixer diameter
k	constant defined by Eq. (*10*)
L	length of mixer blades [m]
n	rotational speed of the mixer [s^{-1}]
Δp_i	pressure reading of the i-th tube hole [$N\ m^{-2}$]
p_{st}	local mean time value of static pressure [$N\ m^{-2}$]
P_{st}	dimensionless, local mean time value of static pressure
q	ratio of angular liquid velocity in rotor region of the mixer
r	radial coordinate [m]
W	dimensionless absolute value of vector **W** of local, mean time velocity
$\bar{w}$	absolute value of vector $\bar{\mathbf{w}}$ of local, in time mean velocity [$m\ s^{-1}$]
w'	fluctuation component of local velocity [$m\ s^{-1}$]
W_{ax}	axial component of vector **W**
$\bar{w}_{ax}$	axial component of vector $\bar{\mathbf{w}}$ [$m\ s^{-1}$]
W_{rad}	radial component of vector **W**
$\bar{w}_{rad}$	radial component of vector $\bar{\mathbf{w}}$ [$m\ s^{-1}$]
W_{tg}	tangential component of vector **W**
$\bar{w}_{tg}$	tangential component of vector **w** [$m\ s^{-1}$]
$\dot{V}$	pumping capacity of the mixer [$m^3\ s^{-1}$]
Z	dimensionless vertical (axial) coordinate defined by Eq. (*12*)
z	vertical (axial) coordinate [m]
z_i [$i = 1, 2$]	axial coordinate of bottoms of rotor region [m]
α	angle between the local, mean time velocity vector and the vertical plane passing through the axis of symmetry of the mixed system (deg)
α_2	angle between the axis of Pitot tube and the vertical plane [deg]
β	angle between the local, mean time velocity vector and the horizontal plane [dg]e
β_2	angle between the Pitot tube and the horizontal plane [deg]
η	dynamic viscosity of the charge [$kg\ m^{-1}\ s^{-1}$]
ω	angular velocity of mixer [s^{-1}]
ω'	angular liquid velocity in rotor region [s^{-1}]
ϱ	density of charge [$kg\ m^{-3}$]
σ_N	estimate of standard deviation of quantity N
$K_p \equiv \dot{V}/nd^3$	flow-rate criterion
$Re \equiv nd^2\varrho/\eta$	Reynolds number
$\Delta w/\bar{w}$	relative error of absolute value of local, mean time velocity vector

REFERENCES

1. Rushton J. H., Mack D. E., Everett H. J.: Trans. A.I.CH.E. *42*, 441 (1946).
2. Sachs J. P., Rushton J. M.: Chem. Eng. Progr. *50*, 597 (1954).
3. Metzner A. B., Taylor I. S.: A.I.CH.E. J. *6*, 109 (1960).
4. Cutter L. A.: A.I.CH.E. J. *12*, 35 (1966).
5. Sato T., Taniyama I.: Chem. Eng. (Japan) *29*, 153 (1965).
6. Aiba S.: A.I.CH.E. J. *4*, 485 (1958).
7. Norwood K. W., Metzner A. B.: A.I.CH.E. J. *6*, 432 (1960).
8. Oldshue J. Y.: Chem. Proc. Eng. *47*, 183 (1966).
9. Holmes D. B., Voncken R. M., Dekker J. A.: Chem. Eng. Sci. *19*, 201 (1964).
10. Kafarov V. V., Ogorodnik I. M.: Chim. Neft. Mašinostr. No. 1, 22 (1967).
11. Nagata S., Yamamoto Y., Hashimoto K., Naruse Y.: Mem. Fac. Eng., Kyoto Univ. *21*, 360 (1959).
12. Nagata S., Yamamoto Y., Ujihara M.: Mem. Fac. Eng., Kyoto Univ. *20*, 336 (1958).
13. Blaziński H., Tyczkowski A.: Chem. Stosow. *3B*, 275 (1967).
14. Wolf D., Manning F. S.: Can. J. Chem. Eng. *44*, 137 (1966).
15. Cooper R. G., Wolf D.: Can. J. Chem. Eng. *45*, 197 (1967); *46*, 94 (1968).
16. Van de Vusse J. B.: Chem. Eng. Sci. *4*, 178 (1955).
17. Souza A. D., Pike R. W.: *67-th Nat. Meeting A.I.CH.E. Atlanta 1970.*
18. Kočin N. J., Kibel I. A., Roze N. V.: *Teoretičeskaja Gidromechanika,* Part 1. Fizmatgiz, Moscow 1963.
19. Krátký J., Fořt I., Kroužilová Z., Drbohlav J.: Strojírenství, in press.
20. Kudrna V., Fořt I., Cvilink J., Drbohlav J., Eslamy M.: This Journal *37*, 241 (1972).

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